MASS EXCHANGE BETWEEN A SOLID SPHERICAL BODY AND A CURRENT-CARRYING LIQUID

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The problem of the determination of the action of an electric current on mass exchange in a solid body – liquid system is solved.

It is known that near a solid body immersed in an electrically conducting liquid through which an electric current flows, the liquid will move. The cause of this motion is the action of the magnetic field of the electric current on the current-carrying liquid. Chow [1] determined the velocity field about a spherical particle under conditions of superposition of two slow motions: the forced motion of the particle and the motion of the liquid caused by the electric current. The stream function in this case has the form

$$\psi = \frac{U_0 a^2}{2}\left\{\left[\left(\frac{r}{a}\right)^2 - \frac{3}{2}\left(\frac{r}{a}\right) + \frac{1}{2}\left(\frac{a}{r}\right)\right]\sin^2\theta - K\left[\left(\frac{r}{a}\right)^2 - \frac{5}{2} + \left(\frac{a}{r}\right) + \frac{1}{2}\left(\frac{a}{r}\right)^2\right]\sin^2\theta\cos\theta\right\}, \qquad (1)$$

where $K = \mu_e I_0^2 a^3 / 8\mu U_0$.

Equation (1) with sufficient accuracy is valid when the numbers $Re \ll 1$ and $Re_m \ll 1$. The solution of the formulated hydrodynamic problem established prerequisites for the solution of a problem of mass exchange under the same conditions.

We consider the steady-state transport of matter from the surface of a dissolving sphere into the surrounding liquid. Assuming that the case being considered corresponds to large values of the criterion Pe (which is caused by values of the criterion $Pr = 10^3$-10^6) we will assume that the representations concerning the diffusion layer that arises on the surface of the sphere are correct. The system of differential equations and boundary conditions that determines the concentration of solute $c = c(r, \theta)$ has the form

$$u_r \frac{\partial c}{\partial r} + \frac{u_\theta}{r}\frac{\partial c}{\partial \theta} = D\frac{\partial^2 c}{\partial r^2},$$
$$c(a, \theta) = c_s,\ c(\infty, \theta) = c_0,\ \left.\frac{\partial c}{\partial \theta}\right|_{\theta=0} = 0. \qquad (2)$$

System (2) with the use of Eq. (1) is solved in Mises' variables and leads to the result

$$\frac{c_s - c}{c_s - c_0} = \frac{1}{1{,}16}\int_0^z \exp\left(-\frac{4}{9}x^3\right)dx,$$

where

$$z = 0{,}397\,\frac{(r-a)}{a}\,Pe^{1/3}\frac{\sqrt{7K\cos\theta \pm 3}\sin\theta}{\left(\int_0^\theta \sqrt{7K\cos x \pm 3}\sin^2 x\,dx\right)^{1/3}}.$$

The sign will be positive if the directions of motion in the region being considered coincide.

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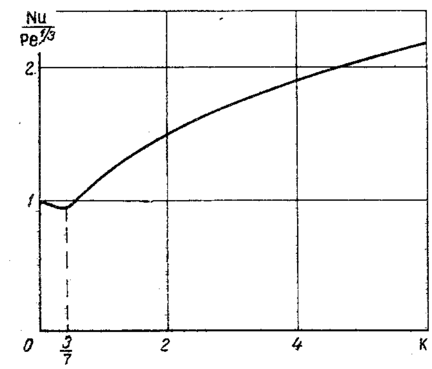

Fig. 1. Effect of electric current on dimensionless dependence determining the mass-transfer coefficient.

For K > 3/7 the mass flow from the entire surface of the sphere per unit time equals

$$J = 2\pi a\, 0.342\, D\,(c_s - c_0)\,\mathrm{Pe}^{1/3}\left[\int\limits_0^{\frac{\pi}{2}+\arcsin\frac{3}{7K}} \frac{\sqrt{7K\cos\theta + 3}\,\sin^2\theta\, d\theta}{\left(\int\limits_0^{\theta}\sqrt{7K\cos x + 3}\,\sin^2 x\, dx\right)^{1/3}}\right.$$

$$\left. + \int\limits_0^{\frac{\pi}{2}-\arcsin\frac{3}{7K}} \frac{\sqrt{7K\cos\theta - 3}\,\sin^2\theta\, d\theta}{\left(\int\limits_0^{\theta}\sqrt{7K\cos x - 3}\,\sin^2 x\, dx\right)^{\frac{1}{3}}}\right].$$

Hence,

$$\mathrm{Nu} = 0.4825\,\mathrm{Pe}^{1/3}\left\{\left[\sqrt{\frac{7K}{3}+1}\left[\frac{2\,(p_1^4 - p_1^2 + 1)}{p_1^3}E\left(\frac{1}{p_1}\right) - \frac{(p_1^2-1)(2p_1^2-1)}{p_1^3}F\left(\frac{1}{p_1}\right)\right]\right]^{2/3} + \left[\sqrt{\frac{7K}{3}-1}\left[\frac{2\,(p_2^4 - p_2^2 + 1)}{p_2^3} \times E\left(\frac{1}{p_2}\right) - \frac{(p_2^2-1)(2p_2^2-1)}{p_2^3}F\left(\frac{1}{p_2}\right)\right]\right]^{2/3}\right\}, \tag{3}$$

where $p_1^2 = 14K/(7K+3)$; $p_2^2 = 14K/(7K-3)$. For large K with accuracy to $O(1/K)$ from (3) we have

$$\mathrm{Nu} = 1.2\,\mathrm{Pe}^{1/3}\,K^{1/3}.$$

For $U_0 = 0$ we have $K = \infty$ and, hence,

$$\mathrm{Nu} = 1.2\left(\frac{\mu_e I_0^2 a^4}{8\mu D}\right)^{1/3}.$$

If $K \le 3/7$, then

$$\mathrm{Nu} = 0.4825\,\mathrm{Pe}^{1/3}\left(\frac{7K}{3}+1\right)^{1/3}\left[\frac{2\,(p_1^4 - p_1^2 + 1)}{p_1^4}E(p_1) - \frac{(1-p_1^2)(2-p_1^2)}{p_1^4}F(p_1)\right]^{2/3}. \tag{4}$$

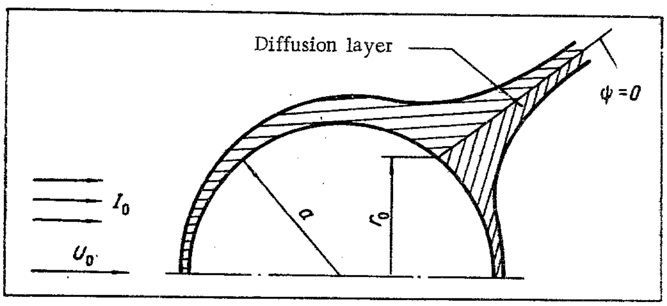

Fig. 2. Distribution of thickness of diffusion layer over sphere surface for K > 3/7.

For $I_0 = 0$ we have $K = 0$ and

$$Nu = 0.991\, Pe^{1/3},$$

which coincides with the solution assumed in [2]. The results (3) and (4) are represented in Fig. 1.

There is definite interest in the distribution of the thickness of the diffusion layer over the surface of the sphere for $K > 3/7$ (see Fig. 2).

Unlike simple flow, mass is washed away not from the afterbody of the sphere but from the surface of a ring of radius r_0. Thus, the results obtained here indicate the principal possibility of intensifying the mass exchange by using a conducting liquid.

NOTATION

r, θ, and φ	are the spherical coordinates;
a	is the sphere radius;
c	is the concentration of the solute;
c_0	is the concentration of the solute far from the sphere;
c_s	is the concentration of the solute at the sphere surface;
U_0	is the velocity of the current-carrying liquid far from the sphere;
u_θ	is the tangential component of the velocity of the liquid;
u_r	is the radial component of the velocity of the liquid;
I_0	is the electric-current density far from the sphere;
τ	is the electrical conductivity of the liquid;
μ_l	is the magnetic permeability of the liquid;
μ	is the dynamic viscosity;
ν	is the kinematic viscosity;
ψ	is the stream function;
x	is the variable of integration;
k	is the mass-transfer coefficient;
D	is the diffusion coefficient;
J	is the flow from the sphere surface per unit time;
$Nu = k2a/D$	is the diffusional Nusselt number;
$Pr = \nu/D$	is the diffusional Prandtl number;
$Pe = U_0 2a/D$	is the diffusional Peclet number;
$Re = U_0 2a/\nu$	is the Reynolds number;
$Re_m = \mu_e \tau U_0 2a$	is the magnetic Reynolds number;
F	is the complete elliptic integral of the first kind;
E	is a complete elliptic integral of the second kind;
$K = \mu_e I_0^2 a^3 / 8\mu U_0$	is a dimensionless parameter.

LITERATURE CITED

1. C. Y. Chow, Phys. Fluids, 9, 933 (1966).
2. G. A. Aksel'rud, Zh. Fiz. Khim., 27, 1446 (1953).